

University of California, Berkeley  
Physics H7C Fall 1999 (*Strovink*)

## PROBLEM SET 2

### 1.

A closely spaced circular parallel plate capacitor with long axial leads has a small (temporarily constant) current  $I$  passing through it (because the voltage across it is changing very slowly). The plates are perfectly conducting and have radius  $b$ .

(a.)

Use Gauss's law to find the time rate of change  $dE/dt$  of the electric field within the plates. You may assume that the charge densities on the inside surfaces of the plates do not vary appreciably across those surfaces.

(b.)

Use the Ampère-Maxwell equation to find the magnitude and direction of the magnetic field halfway between the plates, at a radius  $r < b$  from the axis.

(c.)

Use Ampère's law to evaluate the magnetic field in the vicinity of one of the long axial leads, far from the capacitor. Compare it to the answer for (b.).

(d.)

Suppose instead that  $I$  varies slowly. Far from the fringe of the capacitor, would you expect the electric field to vary slightly with  $r$ ? Explain.

### 2.

(based on *Purcell 10.14.*)

Consider three closely spaced parallel plate capacitors of the same square area and plate separation. The first capacitor  $C_1$  consists only of those plates and vacuum. Both  $C_2$  and  $C_3$  are half-filled with an insulating material having dielectric constant  $\epsilon$ , but the dielectric is arranged in different ways:  $C_2$ 's dielectric extends over the full plate area, but fills only the half gap closest to one of the plates;  $C_3$ 's dielectric extends from one plate to the other, but covers only half of the gap area. (The dielectric boundaries are always either parallel or perpendicular to the plates.)

Calculate the capacitances  $C_2$  and  $C_3$ , expressed as a ratio to  $C_1$ .

### 3.

(based on *Purcell 10.23.*)

Consider an oscillating electric field,  $E_0 \cos \omega t$ , inside a dielectric medium that is not a perfect insulator. The medium has dielectric constant  $\epsilon$  and conductivity  $\sigma$ . This could be the electric field of some leaky capacitor which is part of a resonant circuit, or it could be the electric field at a particular location in an electromagnetic wave. Work in SI units. Show that the  $Q$  factor, defined by

$$Q = \omega \frac{\text{energy stored}}{\text{average power dissipated}} ,$$

is  $\epsilon\omega/\sigma$  for this system, and evaluate it for seawater at a frequency of 1000 MHz. The conductivity is  $4 \text{ (ohm-m)}^{-1}$ , and the dielectric constant may be assumed to be the same as that of pure water at the same frequency,

$$\frac{\epsilon}{\epsilon_0} \approx 78 .$$

What does your result suggest about the propagation of decimeter waves through seawater?

### 4.

(based on *Purcell 10.24.*)

A block of glass, refractive index  $n = \sqrt{\epsilon/\epsilon_0}$ , fills the space  $y > 0$ , its surface being the  $xz$  plane. A plane wave traveling in the positive  $y$  direction through the empty space  $y < 0$  is incident upon this surface. The electric field in this wave is  $\hat{\mathbf{z}} E_i \sin(ky - \omega t)$ . There is a wave inside the glass block, described exactly by

$$\begin{aligned} \mathbf{E} &= \hat{\mathbf{z}} E_0 \sin(ky - \omega t) \\ \mathbf{B} &= \hat{\mathbf{x}} B_0 \sin(ky - \omega t) . \end{aligned}$$

There is also a reflected wave in the space  $y < 0$ , traveling away from the glass in the negative  $y$  direction. Its electric field is  $\hat{\mathbf{z}} E_r \sin(ky + \omega t)$ . Of course, each wave has its magnetic field of

amplitude, respectively,  $B_i$ ,  $B_0$ , and  $B_r$ . The total magnetic field must be continuous at  $y = 0$ , and the total electric field, being parallel to the surface, must be continuous also. Show that this requirement, and the relation of  $B_0$  to  $E_0$  given in the equation

$$B_0 = \sqrt{\epsilon\mu_0} E_0 \quad ,$$

suffice to determine the ratio of  $E_r$  to  $E_i$ . When a light wave is incident normally on a vacuum-glass interface, what fraction of the energy is reflected if the index  $n$  is 1.6?

### 5.

(based on *Purcell 11.11.*)

Write out Maxwell's equations as they would appear if we had magnetic charge and magnetic charge currents as well as electric charge and electric currents. Invent any new symbols you need and define carefully what they stand for. Be particularly careful about  $+$  and  $-$  signs. Work in SI units.

### 6.

(based on *Purcell 11.17.*)

An iron plate 0.2 m thick is magnetized to saturation in a direction parallel to the surface of the plate. A "10 GeV/c" muon having momentum  $p$ , with  $pc = 10^{10}$  eV, moving perpendicular to the plate's surface, enters the plate and passes through it with relatively little loss of energy. (This is possible because the muon, of mass  $m$  with  $mc^2 \approx 10^8$  eV, is  $\approx 200$  times heavier than an electron, so it radiates  $200^2$  times fewer photons.) Calculate approximately the angular deflection of the muon's trajectory. Take the saturation magnetization of iron to be equivalent to  $1.5 \times 10^{29}$  electron magnetic moments per  $\text{m}^3$  (the electron magnetic moment is  $\mu_B \approx 6 \times 10^{-5}$  eV per Tesla).

### 7.

Fowles 1.4.

### 8.

Fowles 1.6.